四分円内の正方形と正三角形の１辺について　Ⅰ

数実研会員　時岡郁夫

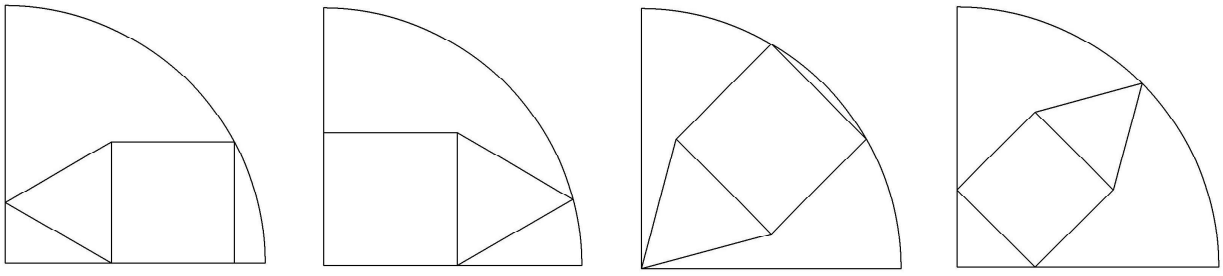
半径１の四分円内に正方形１個と正三角形１個をいろいろな配置で内接させる（53通り）。正方形，正三角形の１辺の長さをそれぞれ a, b とおいて求めてみた。

１．正方形と正三角形の１辺が等しい（ $a=b$ ）のとき（辺を共有する）　8通り
２．正方形と正三角形の１辺が等しい（ $a=b$ ）のとき（辺を共有しない）　14通り
３．正方形の１辺が正三角形の１辺より大（ $a>b$ ）のとき　15通り
４．正方形の１辺が正三角形の１辺より小（ $a<b$ ）のとき　16通り

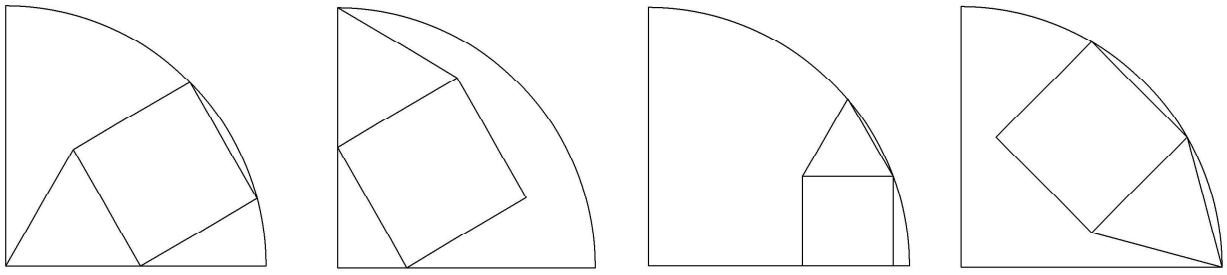
ただし，■印の問題の正方形と▲印の問題の正三角形は，四分円の中心角の二等分線に関して対称，▼印の問題の図形は，四分円の中心角の三等分線（正三角形の角の二等分線）に関して対称である。また，★印の問題は， a または b の値が複雑になるか或いは代数的に求めることができないことが分かった。

１．正方形と正三角形の１辺が等しい（ $a=b$ ）のとき（辺を共有する）　8通り

(1)　(2)　(3) ▲　(4) ■

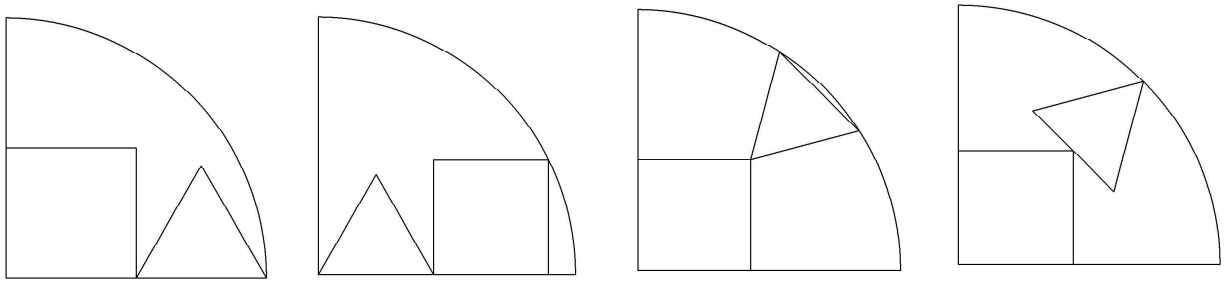


(5)　(6)　(7)　(8)

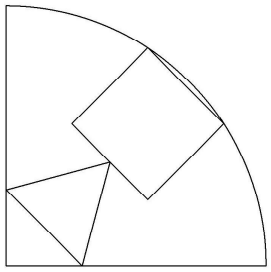


２．正方形と正三角形の１辺が等しい（ $a=b$ ）のとき（辺を共有しない）　14通り

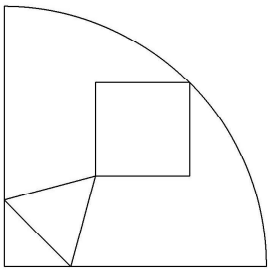
(1)　(2)　(3)　(4) ▲



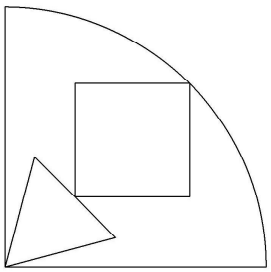
(5) ▲, ■



(6) ▲, ■



(7) ▲, ■



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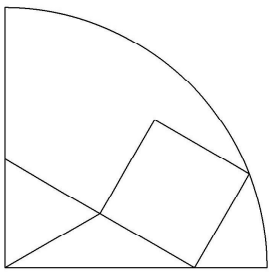
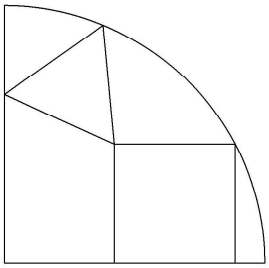


(9) ★



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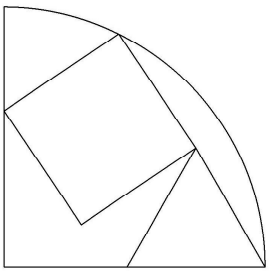
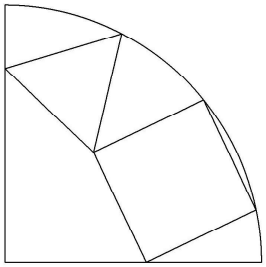


(11) ★



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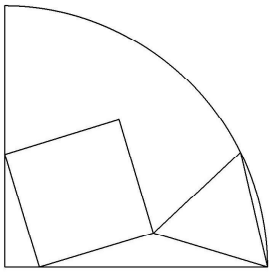
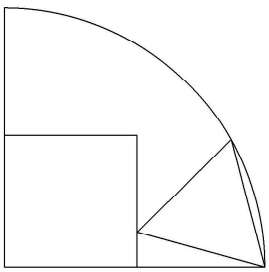
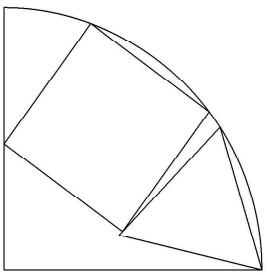


(13) ★

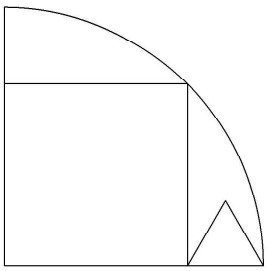


(14) ★

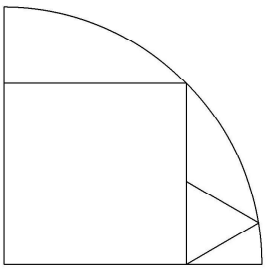


3．正方形の 1 辺が正三角形の 1 辺より大（ $a>b$ ）のとき　15 通り

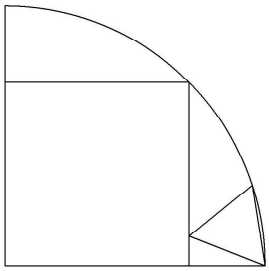
(1)



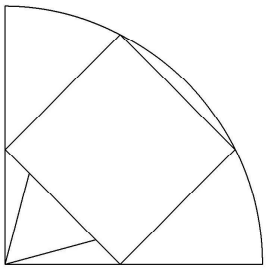
(2)



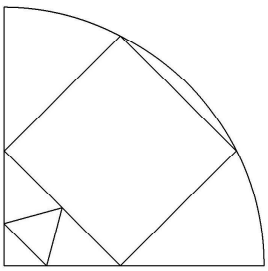
(3)



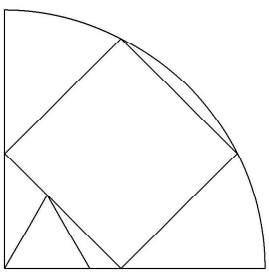
(4)



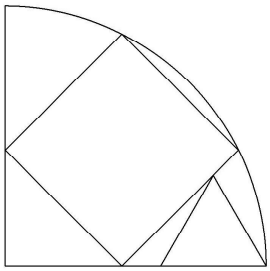
(5) ▲



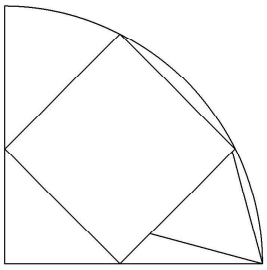
(6)



(7)



(8)



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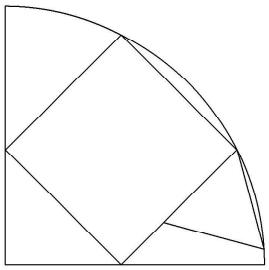
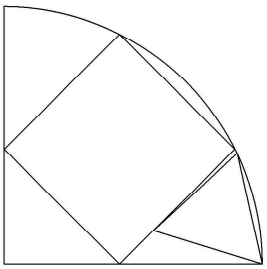
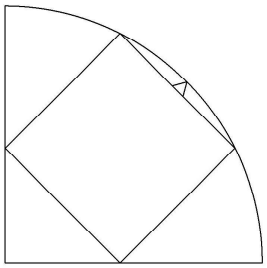


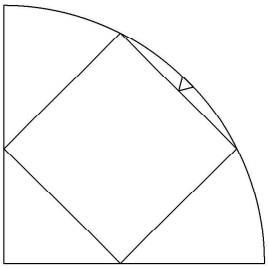
(10)



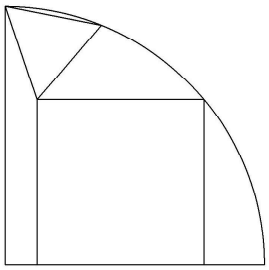
(11) ▲



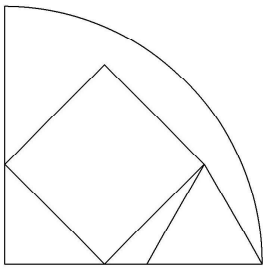
(12) ▲



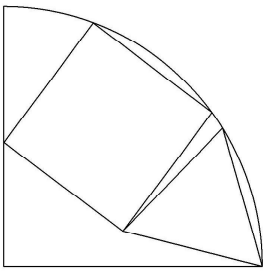
(13) ★



(14) ■

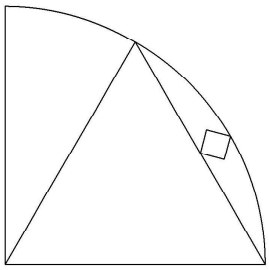


(15)

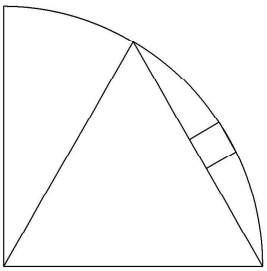


4．正方形の 1 辺が正三角形の 1 辺より小（ $a < b$ ）のとき　16 通り

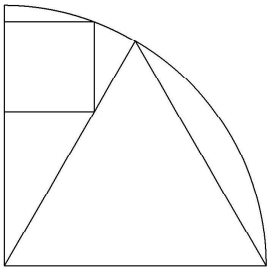
(1) ▼



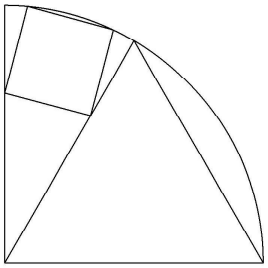
(2) ▼



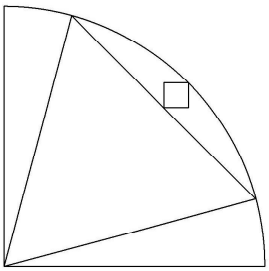
(3)



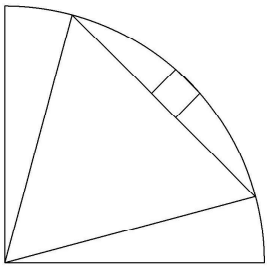
(4)



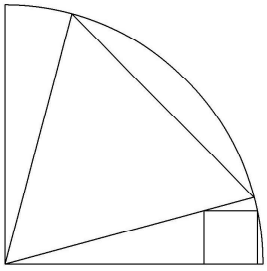
(5) ■



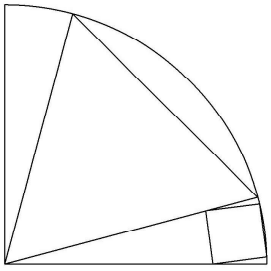
(6) ■



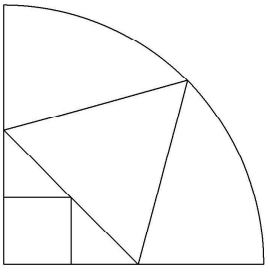
(7)



(8)



(9) ▲



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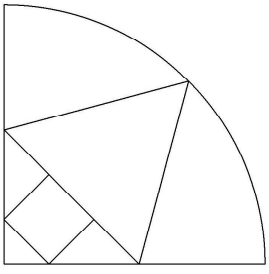

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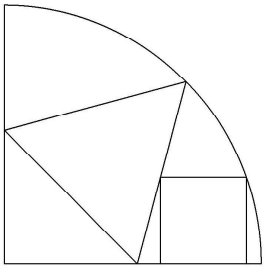

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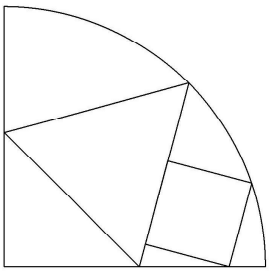
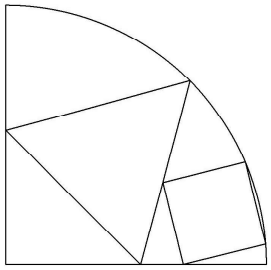
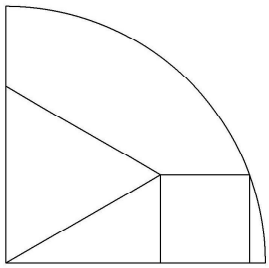
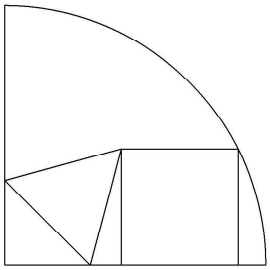
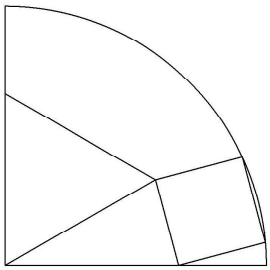


(13) ▲, ★　　(14)　　(15) ▲　　(16)

5．答

1．(1) $2\sqrt{\dfrac{11-4\sqrt{3}}{73}}$　(2) $\dfrac{\sqrt{6}-\sqrt{2}}{2}$　(3) $\dfrac{\sqrt{6}-\sqrt{2}}{2}$　(4) $\dfrac{3-\sqrt{3}}{3}$

(5) $\dfrac{\sqrt{6}-\sqrt{2}}{2}$　(6) $2(2-\sqrt{3})$　(7) $\sqrt{\dfrac{5-2\sqrt{3}}{13}}$　(8) $\dfrac{\sqrt{6}-\sqrt{2}}{2}$

2．(1) $\dfrac{1}{2}$　(2) $\dfrac{\sqrt{5}}{5}$　(3) $\sqrt{\dfrac{3-\sqrt{6}}{3}}$　(4) $\dfrac{2(2\sqrt{2}-\sqrt{3})}{5}$

(5) $2\sqrt{\dfrac{13-6\sqrt{3}}{61}}$　(6) $-5+4\sqrt{2}-3\sqrt{3}+2\sqrt{6}$　(7) $\dfrac{2(2\sqrt{2}-\sqrt{3})}{5}$　(8) $\sqrt{\dfrac{4-\sqrt{3}}{13}}$

(9) $1249a^6-(1334+756\sqrt{3})a^4+(821+484\sqrt{3})a^2-4(41+12\sqrt{3})=0$

$a=\sqrt{\dfrac{\sqrt[3]{c-d}+\sqrt[3]{c+d}+1334+756\sqrt{3}}{3747}}$　（$\fallingdotseq 0.463938$）

（ただし，$c=364290551-774261702\sqrt{3}$，$d=29976\sqrt{1894691580-744533199\sqrt{3}}$）

(10) $\sqrt{3}\,a^3-a^2-(2+\sqrt{3})a+2=0$

$a=\dfrac{1}{3\sqrt{3}}\left[2\sqrt{10+6\sqrt{3}}\cos\left\{\dfrac{2\pi}{3}-\dfrac{1}{3}Arc\cos\left(-\dfrac{\sqrt{-7142857+4123953\sqrt{3}}}{8\sqrt{2}}\right)\right\}+1\right]$

（$\fallingdotseq 0.529693$）

(11) $16-136a^2+381a^4-570a^6+1065a^8-1972a^{10}+1901a^{12}-814a^{14}+121a^{16}=0$

（$a\fallingdotseq 0.474893$）

(12) $\sqrt{\dfrac{17+2\sqrt{3}-\sqrt{17}-2\sqrt{51}}{10}}$

(13) $a^4+2a^3-a^2-8a+4=0$　（$a\fallingdotseq\cdot 0.508969$）

(14) $8a^8-2(23+3\sqrt{3})a^6+3(25+6\sqrt{3})a^4-4(7+\sqrt{3})a^2+2=0$　（$a\fallingdotseq 0.569239$）

3．(1) $a=\dfrac{\sqrt{2}}{2}$，$b=\dfrac{2-\sqrt{2}}{2}$　(2) $a=\dfrac{\sqrt{2}}{2}$，$b=\dfrac{-\sqrt{6}+\sqrt{14}}{4}$

(3) $a=\dfrac{\sqrt{2}}{2}$，$b=\sqrt{\dfrac{5-\sqrt{2}-\sqrt{3+6\sqrt{2}}}{2}}$　(4) $a=\dfrac{\sqrt{10}}{5}$，$b=\dfrac{\sqrt{30}}{15}$

(5) $a=\dfrac{\sqrt{10}}{5}$，$b=\dfrac{\sqrt{30}-\sqrt{10}}{10}$　(6) $a=\dfrac{\sqrt{10}}{5}$，$b=\dfrac{\sqrt{15}-\sqrt{5}}{5}$

(7) $a=\dfrac{\sqrt{10}}{5}$，$b=\dfrac{-5+5\sqrt{3}+\sqrt{5}-\sqrt{15}}{5}$　(8) $a=\dfrac{\sqrt{10}}{5}$，$b=\dfrac{5\sqrt{3}-\sqrt{15}}{15}$

(9) $a=\dfrac{\sqrt{10}}{5}$, $b=\dfrac{3\sqrt{10}-\sqrt{30}}{10}$

(10) $a=\dfrac{\sqrt{10}}{5}$, $b=\sqrt{\dfrac{15-\sqrt{5}+\sqrt{15}-\sqrt{5(1-2\sqrt{3}+10\sqrt{5}+6\sqrt{15})}}{10}}$

(11) $a=\dfrac{\sqrt{10}}{5}$, $b=\dfrac{10\sqrt{3}-3\sqrt{30}}{15}$ (12) $a=\dfrac{\sqrt{10}}{5}$, $b=\dfrac{-3\sqrt{30}+2\sqrt{70}}{20}$

(13) $5a^4-2(1+3\sqrt{3})a^3+(5+2\sqrt{3})a^2+2(-1+\sqrt{3})a-2=0$ ($a\fallingdotseq 0.64251$),
$5b^8-42b^6+(137-8\sqrt{3})b^4+8(-23+4\sqrt{3})b^2+16=0$ ($b\fallingdotseq 0.378309$)

(14) $a=\dfrac{6\sqrt{2}-\sqrt{6}}{11}$, $b=\dfrac{2(-1+2\sqrt{3})}{11}$

(15) $16-352a^2+3144a^4-14976a^6+42325a^8-74966a^{10}+86132a^{12}-66222a^{14}+35437a^{16}-13812a^{18}+3944a^{20}-736a^{22}+64a^{24}=0$ ($a\fallingdotseq 0.575106$), $-32+88b+41b^2-294b^3+192b^4+142b^5-199b^6+40b^7+40b^8-24b^9+4b^{10}=0$ ($b\fallingdotseq 0.556408$)

4. (1) $a=\dfrac{2\sqrt{2}-\sqrt{6}}{4}$, $b=1$ (2) $a=\dfrac{-2\sqrt{3}+\sqrt{17}}{5}$, $b=1$

(3) $a=\sqrt{\dfrac{5-2\sqrt{3}}{13}}$, $b=1$ (4) $a=\sqrt{\dfrac{5-2\sqrt{3}}{13}}$, $b=1$

(5) $a=\sqrt{\dfrac{5-2\sqrt{3}}{13}}$, $b=1$ (6) $a=\dfrac{-2\sqrt{3}+\sqrt{17}}{5}$, $b=1$

(7) $a=\sqrt{\dfrac{13-6\sqrt{3}}{61}}$, $b=1$ (8) $a=\sqrt{\dfrac{10+\sqrt{2}-2\sqrt{3}-3\sqrt{6}}{14}}$, $b=1$

(9) $a=\dfrac{\sqrt{6}-\sqrt{2}}{4}$, $b=\sqrt{3}-1$ (10) $a=\dfrac{\sqrt{6}-\sqrt{2}}{4}$, $b=\sqrt{3}-1$

(11) $a=\dfrac{3\sqrt{2}-8\sqrt{6}+\sqrt{707+331\sqrt{3}}}{61}$, $b=\sqrt{3}-1$

(12) $a=\dfrac{-40+5\sqrt{3}+\sqrt{3(965+436\sqrt{3})}}{122}$, $b=\sqrt{3}-1$

(13) $-2(7+4\sqrt{3})+2(89+50\sqrt{3})a^2-5(97+52\sqrt{3})a^4+(529+248\sqrt{3})a^6-(251+74\sqrt{3})a^8+49a^{10}=0$
($a\fallingdotseq 0.325942$), $b=\sqrt{3}-1$

(14) $a=\sqrt{\dfrac{5-2\sqrt{3}}{13}}$, $b=2\sqrt{\dfrac{5-2\sqrt{3}}{13}}$ (15) $a=\dfrac{\sqrt{5}}{5}$, $b=\dfrac{\sqrt{30}-\sqrt{10}}{5}$

(16) $a=\sqrt{\dfrac{5-2\sqrt{3}}{13}}$, $b=\sqrt{\dfrac{4+\sqrt{3}}{13}}$

【お願い】他の配置があれば，ご連絡をお願いします。連絡先： tokioka@i4.gmobb.jp

【参考文献】特になし

(2024/11/30)