An Example of the Instruction of Complex Number Mapping through the Use of Decimal BASIC

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Decimal BASIC is a simple, speedy, sophisticated program language. The many functional features of this language cannot be thoroughly explained in this short paper. However, in this report, I will describe the effective use of the program in the instruction of the complex plane in the high school context.

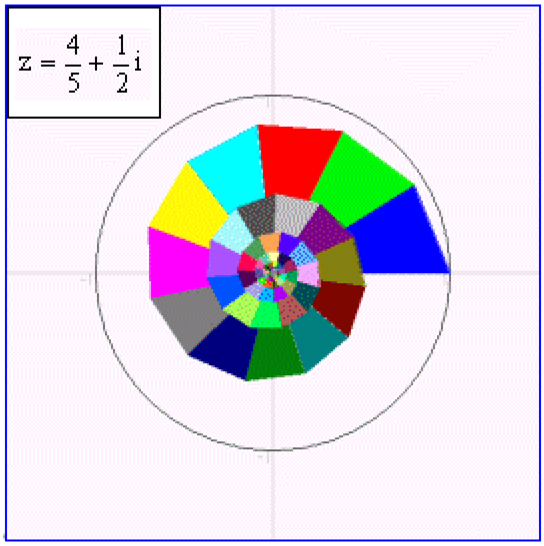
<prog1> is the program to show the complex number z^n. It acquires a complex number value by a function of the mouse using the “get point” direction and it classifies the involution. By the position of the mouse, a vivid image appears, resembling a kaleidoscope of colors.

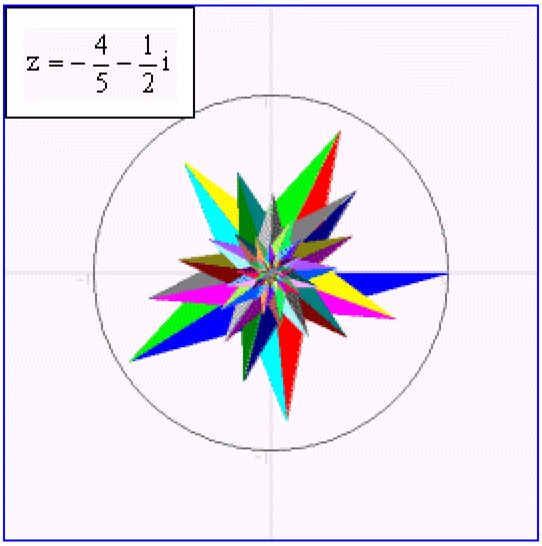
By using Decimal BASIC, the result is quick even if it includes this screen effect.

```
<prog1>
! The show of complex number z^n
let s=1.5
set window -s,s,-s,s
draw axes
option angle degrees
for  t=1  to  360                                   ! The show of unit circle
     plot  lines:cos(t),sin(t);
next  t
get  point:a,b                                      ! The complex number specification by the mouse
print "z=";a;"+";b;"i"
let  c=angle(a,b)
let r=sqr(a^2+b^2)
for  n=1  to  2000                                  ! The show of z^n
    set  area  color  n+1
     let  n1=n-1
    plot  area:0,0;  r^n*cos(n*c),r^n*sin(n*c);  r^n1*cos(n1*c),r^n1*sin(n1*c)
next n
END
```

In the context of a high school mathematics course, the complex plane field is thought to be difficult to teach. Even though it appeared as the substitution field of the matrix, the instruction of the change is difficult. Though the matrix is the operator as linear transformation of the point, it is because the complex number changes a complex number in the complex plane. Therefore, it places a complex number problem with z=x+yi, and finds the value of the real part and the imaginary part as if separating water and oil. The way of solving mechanically is prevailing, which is not a good way. That it is difficult to take a complex number as the image of the change encourages this bad trend.

However, the vividness of the show image by Decimal BASIC wipes out such concerns. In addition to the impact of merely showing an image, by showing a simplified program, the more effective motivation can be shown. Indeed, like <prog1>, the step, the coordinates setting, the show of unit circle, the entry of the numerical value, the argument, the computation of the absolute value, the show is possible to record easily. By reading a program, you can understand the mechanism of the change of the complex number easily at a glance.





Decimal BASIC has a show feature, "the picture definition".

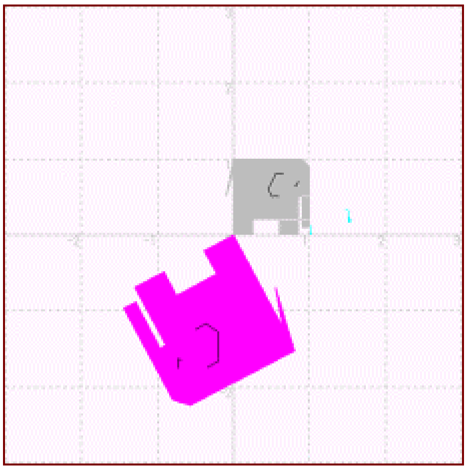
By describing the figure data beforehand as the deputy program, it is possible to transform such as the parallel displacement, the extension, contraction, the turn, and shear. We can think these transformation changes which the complex number as operator has. Therefore, Decimal BASIC has made possible even the change of the set in addition to the point

<prog2> is the program to show the image (the image of the elephant) after the change in taking advantage of the complex number at the elephant (the unit elephant) which was made a square with one piece of length as the big frame.

```
<prog2>
! The exponentiation change of unit elephant
picture    elephant                                                   ! The picture definition of the unit elephant
    set  area  color  col
     plot  area:0,0;0.25,0;0.25,0.2;0.6,0.2;0.6,0;0.85,0;0.85,0.5; &
& 0.9,0.5; 0.9,0.1;1,0.1;1,0.9;0.9,1;0,1;-0.1,0.5;0,0.8;0,0          ! The body of the elephant
     plot  lines:0.65,0.8;0.55,0.8;0.45,0.6;0.5,0.5;0.6,0.506         ! The ear of the elephant
     plot  lines:0.85,0.7;0.82,0.64;0.79,0.66;                        ! The eye of the elephant
end picture

set window -4,4,-4,4
option angle degrees
draw grid
let col=2
draw elephant
input prompt "a,b=":a,b
let    r1=sqr(a^2+b^2)                                                ! The absolute value
let    c1=angle(a,b)                                                  ! The Argument
if r1>1 then
     let  s=0.02
else
     let  s=-0.02
end if
if c1<0 then
      let    c1=360+c1
end if
for  r=1  to  r1  step  s                                             ! The zooming display
    set  draw  mode  hidden
     let  col=0
    draw  elephant  with  scale(r-s)
     draw  grid
     let  col=5
    draw  elephant  with  scale(r)
    set  draw  mode  explicit
next r
for  c=0  to  c1  step  1                                             ! The turn
     set  draw  mode  hidden
     let  col=0
    draw  elephant  with  scale(r)*rotate  (c-1)
     draw  grid
     let  col=7
    draw  elephant  with  scale(r)*rotate(c)
    set  draw  mode  explicit
next c
LET    col=15
draw elephant


END
```



It computes the size and the argument of the complex number, uses "for next" direction, and transforms a diagram as the animation in order of the elasticity, the turn. There is a way of moving a diagram variously, but here, it takes the technique to rewrite again in the background color to erase an afterimage (the elephant).

"set draw mode hidden", "explicit" are the personal special processing of Decimal BASIC which suppresses flickering in case of animation in the direction which draws a figure with the bit map memory, the direction to make it reflect in the screen respectively.

<prog2> actually is a program which replaced the one

created about 10 years ago in N88BASIC as a supplementary teaching aid of linear transformation in the matrix. In N88BASIC, by using the "line" direction, the formula of linear transformation had to be written in each of the segments, which eventually made the program very long with a great number of lines. Decimal BASIC gets rid of the pain of writing such a long program.

In the above-mentioned two programs, actually, you make complex number x+yi correspond to point (x,y) on the Cartesian coordinates and picture the track only. Therefore, the program doesn't show the nature of the complex number and it is displaying the result of the nature only.

In conventional BASIC, this was a limit. However, in Decimal BASIC, the mode to express in the complex number except handling the numerical value as the decimal number is prepared. In this mode, the output of the complex number is shown in the brackets as a pair in the real part and the imaginary part. An imaginary number unit *i* can be handled by the following line:

let i=sqr(-1) (or, it is let i=complex(0,1)),

The use of the complex number mode expands the use of the complex plane unit mainly in high school mathematics B.

<prog3> is the program to output the size, the argument, and the square number of the input complex number. The development of synthetic skills in students can be attempted by analyzing a problem and experimenting with a program, and not only showing an output.

```
<prog3>
! Computation of the complex number
OPTION ARITHMETIC COMPLEX
OPTION ANGLE DEGREES
   input prompt "a,b=": a,b
   let z=complex(a,b)
   print "z=";re(z);"+";im(z);"i"
   print "|z|=";abs(z)
   print "arg(z)=:";arg(z)
   print "z^2=";re(z^2);"+";im(z^2);"i"
END
```

For example, in (prog1), it sets a complex number mode and rewrites the main part of the show of the syntax "for next" as follows,

plot area:0,0;re(z^n),im(z^n);re(z^n1),im(z^n1)

The show by involution is executed in the gauss plane. When you compare agreement of the difference and the output of the program of both, the nature of the complex numbers such as the theorem of De Moivre is understood.

In the following section, frequency problems in mathematics B will be analyzed by Decimal BASIC.

$f : z \rightarrow \frac{1}{z}$ is called moubius change, the guide of this change becomes difficult in the high school scene of the complex number change. It finds that w is an elastic number to $\frac{1}{|z|}$ in the together conjugate complex number $\bar{z}$ from $w = \frac{1}{z} = \frac{\bar{z}}{|z|^2}$. This relation $|w||z| = 1$ is the reflection in the geometry. Therefore, it is possible to think that moubius change is the reflection change of the conjugate complex number.

To unit circle $|z_1| = 1$ on the complex plane, the circle with the radius 1 which center is α is expressed like $z = z_1 + \boldsymbol{a}$. Let's try to find the locus of the point $w = \frac{1}{z_1 + \boldsymbol{a}}$ which changed point z on this circumference in moubius.

<prog4> is the program which shows in the locus. It is the simple one which points the plot by the "for〜next" syntax and it writes an equation of the locus to output into it.

```
<prog4>
!The mobius change
OPTION ARITHMETIC COMPLEX
set window -3,3,-3,3
for t=0 to 2*pi step 0.01                     ! The show of unit circle
    set line color 3
   let z1=complex(cos(t),sin(t))
   plot lines:re(z1),im(z1);
next t
set area color 3
paint 0,0
draw axes

get point: p,q                                ! The setting central of circle
let a=complex(p,q)

for t=0 to 2*pi step 0.001
   let z=complex(cos(t),sin(t))+a             ! The circle which center is a
    let z2=conj(z)                            ! The conjugate complex
    let w=1/z                                 ! reflection
   set line color 2
    plot lines:re(z),im(z)                    ! The show of the locus
   set line color 7
   plot lines:re(z2),im(z2)
   set line color 4
   plot lines:re(w),im(w)
next t
END
```

When dispatching a program, it is expected that the near point to origin fades away from the origin and the point which left the origin approaches the origin. Because the point z on the circumference which is near the origin is moved to the infinite far point, the radius of the transformed circle approximates the infinity and the whole diagram approximates a straight line.

Therefore, if assuming that the straight line is the circle in the wide sense, it finds that the moubius change is the change (circle-to-circle correspondence) which moves circle to circle.

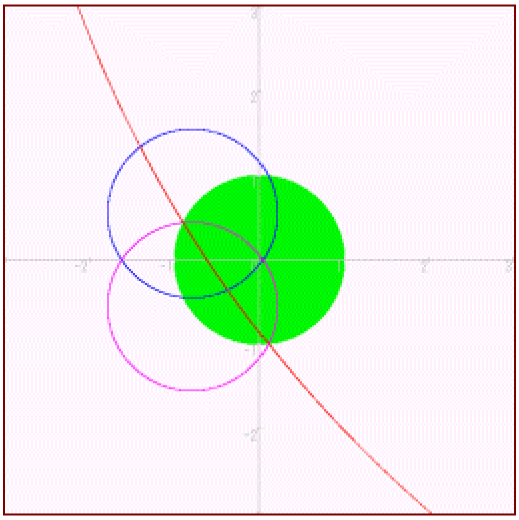
Moreover, the change $f : z \rightarrow \frac{az+b}{gz+d}$ is called complex linear change, this change is circle-to-circle correspondence from resolving $w = \frac{az+b}{gz+d} = \frac{a}{g} + \frac{bg-ad}{g(gz+d)}$. To assemble a program in Decimal BASIC and to confirm this will never be difficult.

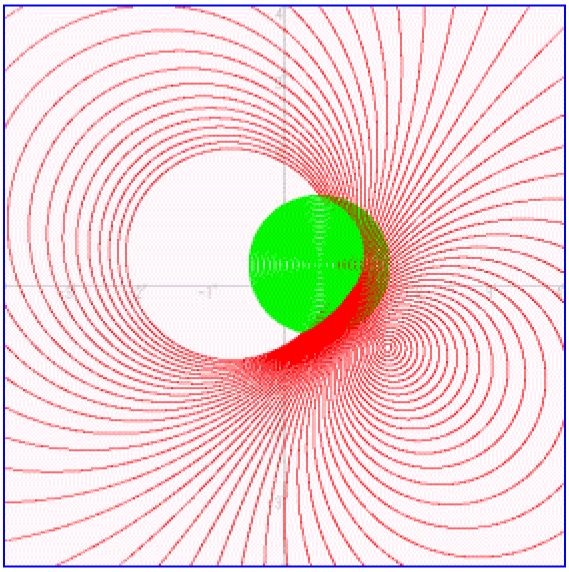
<prog5> is the program to simulate what point the inside of the circle with radius 1 is moved to by the moubius change. In the program, the part of

when exception in～use ～end when

is the phrase of the exception condition. It will be impossible to carry out functions without this line. To have students discover and solve the errors/problems is what mathematics education should strive for.

There is presentation software to show to the student plainly. However, there is little software with which a student himself can experiment and operate to examine the algorithm, estimate and confirm execution. The simplicity of use is one of Decimal BASIC's appealing qualities.





```
<prog5>
! The inner change of circle
set  color  mix(0)  0,0,0                           ! The change of the background color
clear
option arithmetic complex
let s=4
set window -s,s,-s,s
draw axes
input prompt "a+bi=":a,b
set point style 1

for r=1 to 0 step -0.02
   for t=0 to 2*pi step pi/3600
         let  z1=complex(a,b)
         let  z2=complex(r*cos(t),r*sin(t))
         let  z=z1+z2                               ! The definition of circle
         set point color 3
        plot  points:re(z),im(z)                    ! The show of circle
        when  exception  in
             let  w=1/z
            set  point  color  4
             plot  points:re(w),im(w)               ! The show of mobius
         use
         end  when
    next  t
next r

END
```